XXII. *On the Lunar-Diurnal Magnetic Variation at Toronto.*

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In fulfilment of an intention expressed to the Royal Society in November 1853 *, I have now the honour to submit to the Society the results of an investigation into the Moon's diurnal influence on the horizontal and vertical components of the magnetic force at Toronto, and the consequent deduction of the Lunar-diurnal Variations of the Inclination and of the Total Force at that Station.

The processes to which the observations of the Bifilar and of the Vertical force Magnetometers, as received from Toronto, were subjected after their arrival at Woolwich, with a view to this and to other investigations, have been already partially described in a communication presented to the Society in a former part of the present Session †. The processes there described had reference particularly to the reduction of the observations to a uniform temperature of the magnets employed to measure the variations of the respective components of the force,—to the elimination of the larger disturbances,—to the formation of normal values (omitting the disturbances) for each of the components at every hour of mean solar time for periods usually of a month's duration,—and to the deduction of the solar-diurnal variation in different years and different months, after the larger disturbances had been eliminated.

The investigation regarding the Moon's influence was commenced by marking every observation in small figures on the face of the monthly tables with the lunar hour to which each observation most nearly corresponded. This was done in the manner described in the second and third pages of Art. XIX. in the Philosophical Transactions of 1853, when treating of a similar process in the case of the Magnetic Declination. A fresh set of monthly tables were then prepared for every month in each of the five years, in which tables were entered the *differences*, each under the lunar hour to which it most nearly corresponded, between the several observations and their corresponding normals. By this proceeding the diurnal and other variations depending on the time of the solar year, and on the hour of the solar day, were in great part at least eliminated. The differences were marked with a + or a − sign, according as the amount of the force at the time of the observation was greater or less than the monthly normal at the same hour. The mean was then taken in every month of every lunar hour (attending to the signs), and the monthly means were collected into yearly means. The lunar hour to which each observation most

* Philosophical Transactions, 1853, Art. XIX.

† Ibid. 1856, Art. XVI.

nearly corresponded was marked by myself, and the differences from the normals and their arrangement in tables exhibiting the lunar-diurnal influence were prepared, under the superintendence of Mr. MAGRATH, by the Non-commissioned Officers of the Royal Artillery employed in the Woolwich Office. The total number of observations in the five years, after the separation of the larger disturbances, was,—of the Bifilar, 34303; and of the Vertical Force Magnetometer, 31773. The lunar influence at the different lunar hours is shown in the tables in decimals of a scale division; the value of a scale division of the Bifilar being ·000087 parts of the horizontal force, and of the vertical force magnetometer ·000065 parts of the vertical force.

Horizontal Force.—Table I. exhibits, in columns 2 to 6, the mean horary variation of the horizontal force at the different lunar hours in each of the five years ending June 30; and in column 7, the mean of the five years.

TABLE I.

Lunar hours.	In the year ending 30th June. 1844.	1845.	1846.	1847.	1848.	Mean of the five years.	Lunar hours.
1.	2. sc. div.	3. sc. div.	4. sc. div.	5. sc. div.	6. sc. div.	7. sc. div.	8.
0	+0·44	−0·05	+0·10	+0·70	−0·07	+0·22	0
1	+0·48	+0·01	+0·09	+0·70	+0·68	+0·39	1
2	+0·68	+0·59	+0·11	+0·29	+0·42	+0·42	2
3	+0·42	+0·48	+0·71	+0·50	+0·08	+0·44	3
4	+0·58	+0·28	+0·22	+0·09	+0·19	+0·27	4
5	+0·16	+0·68	+0·21	+0·43	+0·74	+0·44	5
6	−0·23	+1·11	−0·03	+0·09	+0·43	+0·27	6
7	−0·13	+0·72	−0·20	−0·25	+0·21	+0·07	7
8	−0·16	+0·59	−0·31	−0·24	−0·19	−0·06	8
9	−0·12	+0·28	−0·23	−0·79	+0·24	−0·12	9
10	−0·43	+0·20	−0·10	−0·62	+0·23	−0·14	10
11	−0·02	+0·32	+0·36	+0·39	−0·22	+0·17	11
12	−0·28	+0·37	+0·15	−0·05	−0·31	−0·02	12
13	+0·11	+0·64	+0·11	+0·23	+0·47	+0·31	13
14	+0·06	+0·84	+0·18	−0·09	+0·40	+0·28	14
15	+0·06	+0·04	+0·23	−0·06	+0·73	+0·20	15
16	0·00	−0·01	+0·13	+0·32	+0·28	+0·14	16
17	−0·17	+0·48	−0·47	0·00	+0·33	+0·03	17
18	−0·08	−0·19	−0·13	−0·24	−0·47	−0·22	18
19	−0·21	−0·13	−0·33	−0·54	−0·80	−0·40	19
20	−0·19	−0·70	−0·11	−0·55	−1·33	−0·58	20
21	−0·19	−0·82	−0·31	−0·09	−1·07	−0·50	21
22	−0·28	−0·44	−0·22	+0·09	−0·65	−0·30	22
23	−0·08	−0·46	−0·15	+0·34	−0·18	−0·11	23

We may represent the values in column 7 of this Table (or the variation of the horizontal force at the several lunar hours on the average of the five years of observation) by the first terms of the usual formula for periodical functions, viz.

$$\Delta_x = A_0 + A_1 \cos a + B_1 \sin a + A_2 \cos 2a + B_2 \sin 2a.$$

By substituting in this formula the numerical values of the coefficients obtained from the numbers in column 7, it becomes

$$\Delta_x = +{\cdot}05 - {\cdot}024 \cos a + {\cdot}214 \sin a + {\cdot}0775 \cos 2a + {\cdot}323 \sin 2a,$$

or the more convenient equivalent expression,

$$\Delta_x = +\cdot 05 + \cdot 215 \sin(a + 353^\circ\cdot 6) + \cdot 3324 \sin(2a + 13^\circ\cdot 5),$$

the coefficients being decimals of a scale-division, and a reckoned in hours, multiplied by 15°, from the time of the Moon's superior culmination. By this formula is obtained the curve which is shown by the stronger line in fig. 1 of the accompanying Plate XIX.; and for the purpose of showing the degree of confidence to which this curve is entitled, as an approximate representation of the variation produced in the horizontal force by the moon in the course of a lunar day, the variation in the different years in columns 2 to 6 of Table I. have been so combined as to form two separate means, one representing the columns headed 1844, 1845 and 1846, and a second representing the columns headed 1846, 1847 and 1848; the years 1844 and 1845 having double weight assigned to them in the first mean, and 1847 and 1848 double weight in the second mean. The formulæ representing these separate means are,—

for 1844 to 1846, $\Delta_x = +\cdot 088 + \cdot 243 \sin(a + 347^\circ\cdot 6) + \cdot 277 \sin(2a + 4^\circ\cdot 4)$,

for 1846 to 1848, $\Delta_x = +\cdot 013 + \cdot 192 \sin(a + 355^\circ\cdot 0) + \cdot 395 \sin(2a + 19^\circ\cdot 2)$.

The curves respectively computed by these formulæ are shown by the fainter lines in fig. 1, in which the stronger line has been already noticed as derived from the mean of the five years.

Vertical Force.—Table II. exhibits in columns 2 to 6 the mean horary variation of the Vertical Force at the different lunar hours in each of the five years ending June 30; and in column 7, the mean variation in the five years.

TABLE II.

Lunar hours.	In the year ending June 30. 1844.	1845.	1846.	1847.	1848.	Mean of the five years.	Lunar hours.
1.	2. sc. div.	3. sc. div.	4. sc. div.	5. sc. div.	6. sc. div.	7. sc. div.	8.
0	+0·08	+0·02	−0·02	−0·03	+0·03	+0·02	0
1	+0·09	−0·03	+0·06	0·00	+0·02	+0·03	1
2	+0·02	−0·07	−0·16	−0·09	−0·18	−0·10	2
3	−0·11	0·00	−0·07	+0·18	−0·06	+0·01	3
4	−0·09	−0·01	−0·01	+0·13	+0·08	+0·02	4
5	+0·01	−0·02	−0·09	+0·07	+0·02	0·00	5
6	−0·12	−0·12	+0·09	+0·01	−0·05	−0·04	6
7	−0·06	−0·24	+0·08	−0·18	−0·02	−0·08	7
8	+0·03	−0·22	+0·03	−0·15	−0·17	−0·10	8
9	−0·04	−0·10	−0·10	+0·02	−0·07	−0·06	9
10	−0·13	−0·10	−0·10	+0·08	−0·11	−0·07	10
11	−0·14	+0·03	−0·03	+0·02	−0·14	−0·05	11
12	−0·08	+0·12	−0·10	+0·01	−0·20	−0·05	12
13	+0·14	+0·03	+0·08	+0·10	−0·16	+0·04	13
14	+0·03	+0·10	+0·04	+0·03	−0·09	+0·02	14
15	+0·08	+0·15	+0·06	+0·02	+0·04	+0·07	15
16	+0·11	+0·18	+0·08	+0·12	+0·06	+0·11	16
17	+0·17	+0·09	+0·04	+0·04	+0·12	+0·09	17
18	+0·19	+0·09	+0·18	−0·03	+0·02	+0·09	18
19	+0·11	+0·05	0·00	0·00	+0·12	+0·06	19
20	+0·14	+0·01	+0·01	−0·14	+0·08	+0·02	20
21	+0·11	−0·07	−0·17	−0·13	+0·05	−0·04	21
22	+0·08	+0·04	−0·21	−0·07	+0·07	−0·02	22
23	+0·11	+0·04	−0·18	−0·13	−0·03	−0·06	23

The curves obtained from the values comprised in this Table are represented to the eye in fig. 2, the fainter lines corresponding to the variations in the separate periods 1844 to 1846, and 1846 to 1848, and the stronger line to the mean of the whole period of five years. The formulæ by which these curves have been computed are as follows:—

$$1844 \text{ to } 1846,\ \Delta_x = +\cdot006 - \cdot092 \sin(a+0^\circ\cdot5) + \cdot036 \sin(2a+345^\circ\cdot1),$$

$$1846 \text{ to } 1848,\ \Delta_x = -\cdot014 - \cdot028 \sin(a+354^\circ\cdot5) + \cdot058 \sin(2a+316\cdot75),$$

$$1844 \text{ to } 1848,\ \Delta_x = -\cdot005 - \cdot058 \sin(a+2^\circ) + \cdot048 \sin(2a+330^\circ).$$

The coefficients are decimals of a scale-division, and a is reckoned in hours (multiplied by 15°) from the epoch of the moon's superior culmination.

Inclination and Total Force.—The lunar-diurnal variations of the Inclination and of the Total force are derived from those of the horizontal and vertical components of the force by the formulæ

$$\Delta\theta = \sin\theta \cos\theta \left(\frac{\Delta Y}{Y} - \frac{\Delta X}{X}\right);$$

$$\frac{\Delta\varphi}{\varphi} = \cos^2\theta \frac{\Delta X}{X} + \sin^2\theta \frac{\Delta Y}{Y};$$

θ being the Inclination, φ the Total force, X its horizontal and Y its vertical components. The variation of the Inclination is expressed in seconds of arc, the + sign implying an increase of the dip of the north end of the magnet. The variation of the Total force is expressed in parts of the total force at Toronto, of which force the approximate absolute value is 13·9 in British units.

Table III. presents the variations of the Inclination and of the Total Force produced by the moon at the different hours of the lunar day, derived from the observed variations of the horizontal and vertical components of the force by the formulæ above stated.

TABLE III.

Lunar hours.	Lunar-diurnal variation of the Inclination.	Lunar-diurnal variation of the Total Force.	Lunar hours.	Lunar-diurnal variation of the Inclination.	Lunar-diurnal variation of the Total Force.	Lunar hours.	Lunar-diurnal variation of the Inclination.	Lunar-diurnal variation of the Total Force.
	″	Parts of the Force.		″	Parts of the Force.		″	Parts of the Force.
0	−0·56	−·0000013	8	+0·05	−·0000051	16	−0·22	+·0000063
1	−1·44	+·0000004	9	+0·17	−·0000058	17	+0·57	+·0000053
2	−2·07	+·0000019	10	0·00	−·0000050	18	+1·31	+·0000033
3	−2·26	+·0000026	11	−0·32	−·0000034	19	+1·92	+·0000006
4	−2·05	+·0000021	12	−0·75	−·0000010	20	+2·14	−·0000020
5	−1·61	+·0000008	13	−1·07	+·0000021	21	+1·93	−·0000031
6	−0·97	−·0000012	14	−1·08	+·0000044	22	+1·33	−·0000035
7	−0·33	−·0000034	15	−0·76	+·0000060	23	+0·42	−·0000026

The values contained in this Table are represented to the eye by the stronger lines in figs. 3 and 4: the two fainter lines in each of these figures exhibiting the values derived from the two half-periods, viz. 1844 to 1846, and 1846 to 1848; the year ending June 30, 1846 having single weight, and the years ending June 30, 1844 and 1845 in the one case, and the years ending June 30, 1847 and 1848 in the other case, having double weight in the respective combinations.

Declination.—To complete the view of the Moon's diurnal influence on the magnetic elements at Toronto, a recalculation has been made of the lunar-diurnal variation of the declination, using the more perfect normals derived by the exclusion of all disturbances equaling or exceeding five minutes of arc*. Table IV. contains the horary variation of the declination at the different hours of the lunar day in each of the six years, from July 1, 1842 to June 30, 1848, and in the eighth column the mean variation in the six years.

Table IV.

One scale-division=0′·721.

Lunar hours.	In the year ending June 30.						Mean of the six years.	Lunar hours.
	1843.	1844.	1845.	1846.	1847.	1848.		
1.	2.	3.	4.	5.	6.	7.	8.	9.
	sc. div.	sc. div.	sc. div.	sc. div.	sc. div.	sc. div.	sc. div.	
0	−0·20	−0·42	−0·45	−0·40	−0·46	−0·37	−0·38	0
1	−0·11	−0·31	−0·18	−0·37	−0·64	−0·29	−0·32	1
2	−0·08	−0·28	+0·04	−0·28	−0·39	−0·31	−0·22	2
3	−0·09	−0·08	−0·07	0·00	−0·36	−0·13	−0·12	3
4	+0·26	+0·09	+0·31	+0·08	+0·13	+0·28	+0·19	4
5	+0·39	+0·09	+0·42	+0·52	+0·20	+0·45	+0·35	5
6	+0·66	+0·40	+0·23	+0·77	+0·15	+0·48	+0·45	6
7	+0·51	+0·29	+0·47	+0·56	+0·31	+0·29	+0·40	7
8	+0·17	+0·26	+0·08	+0·50	+0·09	+0·10	+0·20	8
9	−0·14	+0·21	−0·31	+0·31	−0·23	+0·35	+0·03	9
10	−0·36	−0·24	−0·57	−0·22	−0·40	+0·04	−0·29	10
11	−0·51	−0·33	−0·66	−0·54	−0·24	−0·49	−0·46	11
12	−0·59	−0·48	−0·51	−0·51	−0·52	−0·22	−0·47	12
13	−0·37	−0·27	−0·45	−0·34	−0·29	−0·44	−0·36	13
14	−0·17	−0·31	−0·24	−0·32	+0·07	−0·15	−0·19	14
15	+0·07	−0·14	+0·04	−0·12	+0·52	−0·10	+0·04	15
16	+0·12	+0·22	+0·31	+0·14	+0·43	+0·20	+0·24	16
17	+0·37	+0·45	+0·51	+0·16	+0·55	+0·49	+0·42	17
18	+0·43	+0·74	+0·65	+0·39	+0·47	+0·58	+0·54	18
19	+0·26	+0·25	+0·52	+0·21	+0·55	+0·39	+0·36	19
20	+0·29	+0·15	+0·35	+0·20	+0·22	+0·05	+0·21	20
21	+0·08	+0·15	+0·10	−0·15	−0·21	−0·30	−0·06	21
22	−0·26	−0·05	−0·04	−0·23	−0·42	−0·42	−0·24	22
23	−0·29	−0·37	−0·33	−0·25	−0·31	−0·34	−0·32	23

If we represent the mean of the six years (column 8) by the usual formula of sines and cosines, we have the coefficients of the several terms of the complete formula as follows; the coefficients are expressed in seconds of arc, and a is counted in hours (multiplied by 15°) from the time of the Moon's upper culmination:—

* Philosophical Transactions, 1856, Art. XIX.

Table V.

Arguments ...	Constant	cos a	sin a	cos $2a$	sin $2a$	cos $3a$	sin $3a$	cos $4a$	sin $4a$	cos $5a$	sin $5a$	cos $6a$	sin $6a$
Coefficients	$A_0=$	$A_1=$	$B_1=$	$A_2=$	$B_2=$	$A_3=$	$B_3=$	$A_4=$	$B_4=$	$A_5=$	$B_5=$	$A_6=$	$B_6=$
	0″·0	+0″·20	−1″·03	−19″·18	+0″·45	+1″·84	−0″·32	+1″·21	+0″·59	+0″·61	−0″·48	−0″·22	+0″·57

Arguments ...	cos $7a$	sin $7a$	cos $8a$	sin $8a$	cos $9a$	sin $9a$	cos $10a$	sin $10a$	cos $11a$	sin $11a$	cos $12a$
Coefficients	$A_7=$	$B_7=$	$A_8=$	$B_8=$	$A_9=$	$B_9=$	$A_{10}=$	$B_{10}=$	$A_{11}=$	$B_{11}=$	$A_{12}=$
	−0″·56	+0″·08	+0″·16	−0″·03	−0″·91	−0″·39	−0″·50	−0″·42	+0″·49	+0″·28	−0″·29

The coefficient of principal magnitude is $A_2=-19''\cdot18$, whose argument is cos a. The same coefficient, calculated for the different years, is as follows:—

Year ending June 30, 1843	−19″·35
Year ending June 30, 1844	−17·61
Year ending June 30, 1845	−21·05
Year ending June 30, 1846	−20·22
Year ending June 30, 1847	−19·04
Year ending June 30, 1848	−18·53
And from the mean of the six years . . .	−19·18

Whence we obtain the probable error of $-19''\cdot18$, being the value of A_2 derived from the mean of the six years $=\pm0''\cdot34$.

With the two first terms of this formula, viz.

$$\Delta_x=0''\cdot0+0''\cdot20\cos a-1''\cdot03\sin a-19''\cdot18\cos 2a+0''\cdot45\sin 2a,$$

or its more convenient equivalent,

$$\Delta_x=0''\cdot0-1''\cdot05\sin(a+348°\ 52')+19''\cdot186\sin(2a+271°\ 21'),$$

we obtain the deflections of the north end of the magnet at the several lunar hours as follows:—

Table VI.

Lunar hours.	Deflections.	Lunar hours.	Deflections.	Lunar hours.	Deflections.	Lunar hours.	Deflections.
22	9″·29 to the East	4	9″·19 to the West	10	10″·67 to the East	16	10″·77 to the West
23	15·92	5	15·89	11	17·30	17	17·78
0	18·95	6	18·14	12	19·38	18	20·21
1	16·46	7	15·34	13	16·31	19	17·43
2	9·54	8	8·20	14	8·86	20	10·19
3	0·14	9	0·42 to the East	15	1·04 to the West	21	0·42

Comparing these values with the actual deflections, we find the probable error at each observation hour $\pm1''\cdot37$.

In fig. 5 the darker line represents the deflections in Table VI., constituting the lunar-diurnal variation derived from the mean of the six years; and, for the purpose of showing the accordance of the results when the whole period of six years is divided into three portions, each consisting of two years, viz. July 1842 to June 1844, July 1844 to June 1846, and July 1846 to June 1848, the curves for each of those periods have been computed by the subjoined formulæ, obtained from the values in Table IV., and are represented by the fainter lines in fig. 5. The formulæ are,—

$$1842 \text{ to } 1844\ \Delta_x = +0''{\cdot}41 - 2''{\cdot}09 \sin(a+291^\circ) - 18''{\cdot}1 \sin(2a+87^\circ{\cdot}7),$$

$$1844 \text{ to } 1846\ \Delta_x = +0''{\cdot}30 + 3''{\cdot}04 \sin(a+78^\circ{\cdot}7) + 20''{\cdot}6 \sin(2a+270^\circ{\cdot}1),$$

$$1846 \text{ to } 1848\ \Delta_x = -0''{\cdot}58 - 5''{\cdot}23 \sin(a+53^\circ{\cdot}5) + 18''{\cdot}9 \sin(2a+276^\circ{\cdot}1).$$

The number of hourly observations employed in the investigation of which this paper contains the results, is, of the Declination 40543; of the Horizontal Force 34303; and of the Vertical Force 31773; making in the whole 106,619 observations.

General Conclusions.—The three magnetic elements concur in showing that the moon exercises a sensible magnetic influence at the surface of the earth, producing in every lunar day a Variation in each of the three elements, which is distinctly appreciable by the instruments employed in the Observatories established to carry out the system of observations recommended by the Royal Society, when due care has been taken in conducting the observations, and suitable methods are adopted for elaborating the results.

The Variation in each of the three elements constitutes a double progression in each lunar day: the declination has two easterly and two westerly maxima in the interval between two successive passages of the moon over the astronomical meridian; and the inclination and the total force have each two maxima and two minima due to the moon's action in the same interval; the variation passing in every case four times through zero in the lunar day. The easterly maxima of the horizontal deflection of the north end of the magnet synchronise with the moon's superior and inferior passages of the meridian; the westerly maxima with the lunar hours of 6 and 18. The maxima of the increased magnetic force due to the moon's action occur about the lunar hours of 3 and 16, and the minima about the hours of 9 and 20. The maxima of the inclination, *i.e.* of the dip of the north end of the magnet, occur about the lunar hours of 3 and 14, and the minima about 9 and 20. The extent of the Variation in the lunar day, or the range between the extremes that are widest apart, is in the Declination 38″·33, in the Inclination 4″·4, and in the total Force ·000012 parts of the whole terrestrial magnetic force at Toronto. These are the values derived from the whole period of observation, *i.e.* from six years of the Declination and from five years of the Inclination and total Force. When the whole period is subdivided into two half-periods, the hours of maxima and minima and the extent of the range accord with the results of the whole period in each of the three elements, with slight and wholly insignificant exceptions. The reality of the variations is thus attested no less by the accordance of the results when the whole period during which the phenomena were observed is subdivided into separate and independent portions, than by the systematic character which the Variation is seen to possess when the strictly independent results at the several lunar hours are brought together and exhibited continuously.

As it happens that in the declination the variation resulting from the moon's action is greater, relatively to the instrumental means for measuring it, than either in the inclination or in the total force, it is reasonable to conclude that we have a better opportunity of judging of the particular nature and character of the noon's magnetic influence, by studying the effects produced on the declination than those produced on either of the other elements.

Referring to the Table (Table IV. p. 503) which exhibits the coefficients of all the twelve terms in the formula of sines and cosines by which the results of observation are strictly represented, we perceive that the coefficient of the cosine of twice the hour-angle is not only the one of greatest account, but is in fact the only one which we can with confidence regard as possessed of a substantial value. All the other coefficients are, without exception, not only extremely small in comparison with the one above noticed, but are so small, that they may well be supposed to represent such small deviations from a natural law as may well be ascribed to errors which cannot wholly be extinguished in averages derived from not more than six years of observation. On the other hand, the coefficient in the second term has a value far beyond any explanation resting on the supposition of errors of observation. The probable error at any single hour is $1''\cdot37$, whilst the range of the variation is not less than $38''$. Whilst, therefore, the general result of this investigation is to establish conclusively the existence of a lunar-magnetic influence sensible at the surface of the earth, the lunar-diurnal variation which is thus manifested appears to be consistent with the hypothesis that the moon's magnetism may be, in great part at least if not wholly, derived by induction from the magnetism of the earth.

It is further observable, that in the lunar-diurnal variation there is no appearance of the *decennial* period which constitutes so marked a feature in the solar-diurnal variations.

Lunar-diurnal Variations of the Magnetic Elements.